Generalized Bogoliubov Transformation, Fermion Field and Casimir Effect in a Box

H. Queiroz[a] , J. C. da Silva[a,b], F.C. Khanna[c,d], M. Revzen[e], A. E. Santana[a].

[a] Instituto de Física, Universidade Federal da Bahia,

Campus de Ondina, 40210-340, Salvador, Bahia, Brasil.

[b] Centro Federal de Educação Tecnológica da Bahia

Rua Emídio Santos, 40000-900, Salvador, Bahia, Brasil

[c] Physics Department, Theoretical Physics Institute,

University of Alberta, Edmonton, Alberta T6G 2J1 Canada

[d] TRIUMF, 4004, Westbrook mall, Vancouver, British

Columbia V6T 2A3, Canada

[e] Depart. of Physics, Technion - Institute of Technology

Haifa,32000, Israel.

(February 1, 2008)

Abstract

In this work a generalization of the Bogoliubov transformation is developed to describe a space compactified fermionic field. The method is the fermion counterpart of the formalism introduced earlier for bosons (J. C. da Silva, A. Matos Neto, F.C. Khanna and A.E. Santana, Phys. Rev. A 66 (2002) 052101), and is based on the thermofield dynamics approach. We analyze the energy-momentum tensor for the Casimir effect of a free massless fermion system in a N-dimensional box in contact with a heat bath. As a particular situation we calculate the Casimir energy and pressure for the field in a 3-dimensional box of sizes L_1, L_2, L_3. One interesting result is that the attractive or repulsive nature of the Casimir pressure can change depending on the rate

among L_1, L_2, L_3. This effect is exemplified in the case of $L_1 \to \infty$, and $L_3 = L, L_2 = 0.1L$.

I. INTRODUCTION

In a recent work [1] a generalization of the Bogoliubov transformation was introduced to treat confined boson field in space coordinates at finite temperature. The formalism is based on the thermofield dynamics (TFD) approach [2–8] and has been applied to derive different aspects of the Casimir effect for the electromagnetic field confined between two plates. In this context one interesting physical result is brought about: the Casimir effect is described by a process of condensation, thus shedding a new ingredient on the nature of the quantum vacuum. In this paper we extend the approach introduced in Ref. [1] to fermion fields, and so we apply it to the Casimir effect of a free massless fermionic field in a 3-dimensional box at finite temperature.

The Casimir effect was first proposed by taking into account the effect of the vacuum fluctuation of the electromagnetic field confined within two plates with separation L, using the Dirichlet boundary conditions. The result was an attractive force between the plates given by the negative pressure $P = -\pi^2/240L^4$ (we use natural units: $\hbar = c = 1$) [9]. Over the decades the effect has been applied to different geometries and physical conditions, enjoying a remarkable popularity [10–25] and raising interest, in particular, in the context of microelectronics [26,27].

The effect of temperature was first studied by Lifshitz [28,29] who presented an alternative derivation for the Casimir force, including the analysis of the dielectric nature of the material between the plates. Actually, the effect of temperature on the interaction between the conducting parallel plates may be significant for separations greater than $3\mu m$ [30–35]. For this physical situation of plates, the full analysis of the thermal energy-momentum tensor of the electromagnetic field was carried out by Brown and Maclay [36], performing the calculation of the Casimir free-energy by using the Green's function (the local formulation)

written in a conformally invariant way [17,37,38]. One of our proposal here is to derive the fermionic counterpart of the Brown and Maclay's formula, but for a more general situation of confinement: we consider not only the two plates, but also the case of confinement within an N-dimensional box.

Casimir effect for a massless fermionic field is of great interest in considering the structure of proton in particle physics; thus its physical appeal. In particular, in the phenomenological MIT bag model [39], quarks are confined in a small space region in such a way that there is no fermionic current outside that region. The fermion field then fulfills the so-called bag model boundary condition. The Casimir effect in such a small region, of order $1.0 fm$, is important to define the process of deconfinement in a heavy ion collision at Relativistic Heavy Ion Collider (RHIC), giving rise to the quark-gluon plasma [40]. The gluon field contribution for the Casimir effect is, up to the color quantum numbers, the same as for the electromagnetic field. For the quark field, the problem has been often addressed only by considering the case of two parallel plates [41–46]. Actually, as first demonstrated by Johnson [47], for plates, the fermionic Casimir force is attractive as in the case of the electromagnetic field. On the other hand, depending on the geometry of the confinement, the nature of the Casimir force can change. This is the case, for instance, of a sphere and the Casimir-Boyer model, using mixed boundary conditions for the electromagnetic field, such that the force is repulsive [48–53]. Therefore, the analysis considering fermions in an Euclidian wave-guide (confinement in two-dimensions) and in a 3-dimensional box (confinement in 3-dimensions) may be of interest. We avoid here the approach based on the sum of quantum modes, that was so important in the calculation of the Brown and Maclay. Using, alternately, the method developed in Ref. [1], we perform the calculation of the non-trivial problem of the quark field in a finite volume with the compactification in an N-dimensional box, at finite temperature.

In order to proceed with, we have to adapt the methodology of calculations discussed earlier [1], to encompass a generalized Bogoliubov transformation for fermion fields with the MIT bag model boundary condition; equivalent to an antiperiodic boundary condition [42].

3

This is presented in Section 2, where we derive the energy-momentum tensor for the fermion field at $T \neq 0$, resulting, as an example, in the Stefan-Boltzmann law. This is the usual calculation of TFD for fermions; however it can also be interpreted as a confinement in the time axis, in such a way that the field is under anti-periodic boundary conditions. Using then this methodology for a Euclidian geometry, we can envisage a space compactification. (In fact, this possibility has also been explored in the context of the Matsubara formalism [54,55].) The usual expression of the Casimir effect is thus calculated, considering a proper modified Bogoliubov transformation which will describe the confinement in the z-axis. The familiarity with this kind of calculation for these already known results will suggest to us a general form for a Bogoliubov transformation to describe space compactification in arbitrary dimensions; a subject developed in Section 3. In Section 4, we consider applications of the tensor derived in Section 3. Then we present the main result of this paper: the conformally invariant expression of the thermal energy-momentum tensor describing the Casimir effect of a massless fermionic field confined in a 3-dimensional box. Some particular situations are analyzed, resulting for instance that the attractive or repulsive nature of the Casimir force can change in accordance with the rate among the sizes of the box. This situation is explicitly treated considering the case of wave guide obtained from a box of sizes L_1, L_2, L_3, such that $L_1 \rightarrow \infty$, and $L_3 = L, L_2 = 0.1L$. In Section 5 our concluding remarks are presented.

II. BOGOLIUBOV TRANSFORMATION AND COMPACTIFICATION

The general approach to be used here can be addressed through the following prescription, taken as a generalization of the TFD formalism [1–3] . Given an arbitrary set of operators, say $\mathcal{V}$, with elements denoted by $A_i, i = 1, ..., n$, there exists a mapping describing a doubling in the degrees of freedom defined by $\tau : \mathcal{V} \rightarrow \mathcal{V}$, denoted by $\tau A \tau^{-1} = \widetilde{A}$, satisfying the following conditions

$$(A_i A_j)\widetilde{} = \widetilde{A}_i \widetilde{A}_j, \tag{1}$$

4

$$(cA_i + A_j)\widetilde{} = c^* \widetilde{A}_i + \widetilde{A}_j, \tag{2}$$

$$(A_i^\dagger)\widetilde{} = (\widetilde{A}_i)^\dagger, \tag{3}$$

$$(\widetilde{A}_i)\widetilde{} = A_i. \tag{4}$$

These properties are called *the tilde (or dual) conjugation rules* in TFD. The doubling in the Hilbert space has a new vacuum denoted by $|0, \widetilde{0}\rangle$. Consider $\alpha = (\alpha_0, \alpha_1, \alpha_2, \alpha_3, ...)$ a set of *c*-numbers associated with macroscopic parameters of the system as such: temperature $(\beta = 1/T)$, and three possible parameters describing the spatial confinement (for instance the sizes L_1, L_2 and L_3 of a box in space). Then there exists a Bogoliubov transformation given by

$$B(\alpha) = \begin{pmatrix} u(\alpha) & -v(\alpha) \\ v(\alpha) & u(\alpha) \end{pmatrix}, \tag{5}$$

with $u^2(\alpha) + v^2(\alpha) = 1$.

For an arbitrary operator in $\mathcal{V}$, we use the doublet notation [2]

$$(A^a) = \begin{pmatrix} A(\alpha) \\ \widetilde{A}^\dagger(\alpha) \end{pmatrix} = B(\alpha) \begin{pmatrix} A \\ \widetilde{A}^\dagger \end{pmatrix}, \tag{6}$$

$$(A^a)^\dagger = \begin{pmatrix} A^\dagger(\alpha) , & \widetilde{A}(\alpha) \end{pmatrix}. \tag{7}$$

This kind of notation is useful to calculate the propagator for the confined field.

Here we are concerned with the energy-momentum tensor for a massless fermionic field given by [41,56]

$$T^{\mu\nu}(x) = \langle 0|i\overline{\psi}(x')\gamma^\mu \partial^\nu \psi(x)|0\rangle|_{x' \to x} \tag{8}$$

$$= \gamma^\mu \partial^\nu S(x - x')|_{x' \to x} \tag{9}$$

$$= -4i\partial^\mu \partial^\nu G_0(x - x')|_{x' \to x}, \tag{10}$$

where $S(x - x') = -i\langle 0|T[\psi(x)\overline{\psi}(x')]|0\rangle$ and

$$G_0(x) = \frac{-1}{(2\pi)^4} \int d^4k \ e^{-ik \cdot x} G_0(k)$$

$$= \frac{-i}{(2\pi)^2} \frac{1}{x^2 - i\varepsilon},$$

such that

$$G_0(k) = \frac{1}{k^2 + i\varepsilon},$$

and the Minkowski metric with a signature $(+ - --)$. With $T^{\mu\nu}(x)$, we can introduce the confined (α-)energy-momentum tensor $\mathcal{T}^{\mu\nu(ab)}(x; \alpha)$ defined by

$$\mathcal{T}^{\mu\nu(ab)}(x; \alpha) = \langle T^{\mu\nu(ab)}(x; \alpha) \rangle - \langle T^{\mu\nu(ab)}(x) \rangle, \tag{11}$$

where $T^{\mu\nu(ab)}(x; \alpha)$ is a function of the field operators $\psi(x; \alpha)$, $\widetilde{\psi}(x; \alpha)$ and $\langle \cdots \rangle = \langle 0, \widetilde{0}| \cdots |0, \widetilde{0}\rangle$. The physical α-tensor is given by the component $\mathcal{T}^{\mu\nu(11)}(x; \alpha)$. Let us then work out this tensor.

Considering the TFD prescription [2,57], we have

$$S^{(ab)}(x - x') = \begin{pmatrix} S(x - x') & 0 \\ 0 & \widetilde{S}(x - x') \end{pmatrix},$$

with $\widetilde{S}(x - x') = -S^*(x' - x)$. As a result, from Eq.(11), we have

$$\mathcal{T}^{\mu\nu(ab)}(x; \alpha) = -4i\partial^\mu \partial^\nu [G_0^{(ab)}(x - x'; \alpha) - G_0^{(ab)}(x - x')]_{x' \to x}, \tag{12}$$

corresponding to a change in Eq.(9), $S^{(ab)}(x - x')$ by $S(x - x')$. The Green's functions in Eq.(12) are given by

$$G_0^{(ab)}(x - x') = \frac{-1}{(2\pi)^4} \int d^4k \, G_0^{(ab)}(k) \, e^{-ik \cdot (x - x')},$$

where

$$G_0^{(ab)}(k) = \begin{pmatrix} G_0(k) & 0 \\ 0 & G_0^*(k) \end{pmatrix},$$

and the α-counterpart is

$$G_0^{(ab)}(x - x'; \alpha) = \frac{-1}{(2\pi)^4} \int d^4k \, G_0^{(ab)}(k; \alpha) \, e^{-ik \cdot (x - x')}, \tag{13}$$

with

6

$$G_0^{(ab)}(k; \alpha) = B_k^{-1(ac)}(\alpha) G_0^{(cd)}(k) B_k^{(db)}(\alpha),$$

where $B_k^{(ab)}(\alpha)$ is the Bogoliubov transformations given in Eq.(5). Explicitly, the components of $G_0^{(ab)}(k; \alpha)$ are given by

$$G^{11}(k; \alpha) = G_0(k) + v_k^2(\alpha)[G_0^*(k) - G_0(k)],$$

$$G^{12}(k; \alpha) = G^{21}(k; \alpha) = v_k(\alpha)[1 - v_k^2(\alpha)]^{1/2}[G_0^*(k) - G_0(k)],$$

$$G^{22}(k; \alpha) = G_0^*(k) + v_k^2(\alpha)[G_0(k) - G_0^*(k)].$$

The physical quantities are derived from the component $G^{11}(k; \alpha)$.

Let us consider a simple situation in which $\alpha \equiv \beta = 1/T$. In this case $v_k(\beta)$ is defined through the fermion number distribution, that is

$$v_k(\beta) = \frac{e^{-\beta k_0/2}}{[1 + e^{-\beta k_0}]^{1/2}}.$$

Observe that we can write

$$v_k^2(\beta) = \sum_{l=1}^{\infty}(-1)^{l+1}e^{-\beta k_0 l}; \tag{14}$$

leading to the thermal Green's function,

$$G_0^{11}(k; \beta) = G_0(k) + \sum_{l=1}^{\infty}(-1)^{l+1}e^{-\beta k_0 l}[G_0^*(k) - G_0(k)].$$

Using this result in Eq.(13) we derive

$$G_0^{11}(x - x'; \beta) = G_0(x - x') + \sum_{l=1}^{\infty}(-1)^{l+1}[G_0^*(x' - x - i\beta l\widehat{n}_0) - G_0(x - x' - i\beta l\widehat{n}_0)],$$

where $\widehat{n}_0 = (1, 0, 0, 0)$ is a time-like vector. Therefore, from Eq.(12), we find

$$\mathcal{T}^{\mu\nu(11)}(\beta) = 4i\sum_{l=1}^{\infty}(-1)^{l+1}\partial^\mu\partial^\nu[G_0(x' - x + i\beta l\widehat{n}_0) + G_0(x - x' - i\beta l\widehat{n}_0)]|_{x'\to x}.$$

Performing the covariant derivatives, this expression reads

$$\mathcal{T}^{\mu\nu(11)}(\beta) = \frac{8}{(2\pi)^2}\sum_{l=1}^{\infty}(-1)^l\left[\frac{2g^{\mu\nu} - 8\widehat{n}_0^\mu\widehat{n}_0^v}{(\beta l)^4}\right]. \tag{15}$$

7

Well known results for thermal fermionic fields can be derived from this tensor. For instance, the internal energy is given by $E(T) = \mathcal{T}^{00(11)}(\beta)$, that is,

$$E(T) = \frac{7}{4}\frac{\pi^2}{15}T^4,$$

where we have used the Riemann zeta-function [41]

$$\varsigma(4) = \sum_{l=1}^{\infty}(-1)^l\frac{1}{l^4} = -\frac{7}{8}\frac{\pi^4}{90}.$$

As another application, we derive the Casimir effect, by following the above calculations. In this case, instead of Eq.(14), we write $\alpha = i\alpha_3 = ia$

$$v_k^2(a) = \sum_{l=1}^{\infty}(-1)^{l+1}e^{-ik_3al},$$

and use $\widehat{n}_3 = (0,0,0,1)$ a space-like vector. As a consequence we derive

$$\mathcal{T}^{\mu\nu(11)}(a) = \frac{8}{(2\pi)^2}\sum_{l=1}^{\infty}(-1)^l\left[\frac{2g^{\mu\nu} + 8\widehat{n}_3^{\mu}\widehat{n}_3^{\nu}}{(al)^4}\right]. \tag{16}$$

Resulting in a Casimir energy and pressure given, respectively, by

$$E_c(a) = \mathcal{T}^{00(11)}(a) = -\frac{7}{4}\frac{\pi^2}{45a^4},$$

$$P_c(a) = \mathcal{T}^{33(11)}(a) = -3\frac{7}{4}\frac{\pi^2}{45a^4}, \tag{17}$$

here $a = 2L$, where L is the separation of the plates. This is needed (as in the case of bosons) in order to fix the antiperiodic boundary conditions on the propagator, $S(x - x')$.

III. COMPACTIFICATION IN HIGHER DIMENSIONS

In this section we calculate the Casimir effect for the massless fermions within an N-dimensional (space) box at finite temperature. We proceed then by generalizing the results for the temperature and the Casimir effect at zero temperature which were derived in the last section. Supported by those calculations, we consider a generalization of $v(\alpha)$ as given

in Eq.(14) by writing $\alpha \rightarrow \alpha = (\alpha_0, \alpha_1, \alpha_2, ..., \alpha_N)$, $\widehat{n}_0 = (1,0,0,0,...)$, $\widehat{n}_1 = (0,1,0,0,...)$, $\widehat{n}_2 = (0,0,1,0,...)$,..., $\widehat{n}_N = (0,0,0,...,1)$, vectors in the $(N+1)$-dimensional Minkowski space, such that

$$v_k^2(\alpha) = \sum_{l_0,l_1,...,l_N=0'}^{\infty} (-1)^{l_0+l_1+...+l_N+N_c} \exp\{i \sum_{i=0}^{N} \alpha_i l_i k_i\},$$

where N_c is the number of nonzero component of α_μ in set of sum (for instance for each sum of the type $\sum_{l_j=1}$, then $N_c = 1$, for sums of type $\sum_{l_k l_j=1}$, then $N_c = 2$, and so on), and the symbol $0'$ in the sum means that the situation in with $l_0 = l_1 = ... = l_N = 0$ is excluded. Using this $v_k^2(\alpha)$ and the procedure delineated in Section 2, we obtain

$$\mathcal{T}^{\mu\nu(11)}(\alpha) = 4i \sum_{l_0,l_1,...,l_N=0'}^{\infty} (-1)^{l_0+l_1+...+l_N+N_c} \partial^\mu \partial^\nu$$
$$\times [G_0(x' - x + \alpha_0 l_0 \widehat{n}_0 - \sum_{i=1}^{N} a_i l_i \widehat{n}_i) + G_0(x - x' - \alpha_0 l_0 \widehat{n}_0 - \sum_{i=1}^{N} a_i l_i \widehat{n}_i)]|_{x' \rightarrow x},$$

resulting in

$$\mathcal{T}^{\mu\nu(11)}(\alpha) = \frac{-8}{(2\pi)^2} \sum_{l_0,l_1,...,l_N=0'}^{\infty} (-1)^{l_0+l_1+...+l_N+N_c} \left\{ \frac{1}{[\sum_{i=1}^{N}(\alpha_i l_i)^2 - (\alpha_0 l_0)^2]^2} \right.$$
$$\left. \times \left[2g^{\mu\nu} + 8 \frac{\sum_{i,j=1}^{N}(\alpha_i l_i)(\alpha_j l_j)\widehat{n}_i^\mu \widehat{n}_j^\nu + (\alpha_0 l_0)^2 \widehat{n}_0^\mu \widehat{n}_0^\nu}{\sum_{i=1}^{N}(\alpha_i l_i)^2 - (\alpha_0 l_0)^2} \right] \right\}. \tag{18}$$

Notice that the results given by Eqs.(15) and (16) are particular cases of the energy-momentum tensor given by Eq.(18). Another important aspect is that $\mathcal{T}^{\mu\nu(11)}(\alpha)$ is traceless, as it should be.

IV. THERMAL CASIMIR EFFECT IN A BOX

In the case of a 3-dimensional closed box, considering the temperature effect, we have $\alpha_0 = i\beta$, $\alpha_i = 2L_i (i = 1,2,3)$ where L_i stands for the size of the i-th direction of the box, and $N_c = 4$. Using Eq.(18) in a $(3+1)$ Minkowski space, we obtain

$$\mathcal{T}^{\mu\nu(11)}(\alpha) = \frac{-8}{(2\pi)^2} \sum_{l_0,l_1,l_2,l_3=0'}^{\infty} (-1)^{l_0+l_1+l_2+l_3+N_c} \left\{ \frac{1}{[\sum_{i=1}^{3}(2L_i l_i)^2 + (\beta l_0)^2]^2} \right.$$
$$\left. \times \left[2g^{\mu\nu} + 8 \frac{\sum_{i,j=1}^{3}(2L_i l_i)(2L_j l_j)\widehat{n}_i^\mu \widehat{n}_j^\nu - (\beta l_0)^2 \widehat{n}_0^\mu \widehat{n}_0^\nu}{\sum_{i=1}^{3}(2L_i l_i)^2 + (\beta l_0)^2} \right] \right\}. \tag{19}$$

9

For a cubic box with $L_i = L$, for all $i = 1, 2, 3$, the Casimir energy $(E_{cb}(L))$ and the Casimir pressure $(P_{cb}(L))$ along the direction $i = 3$ at zero temperature are, respectively, given by

$$
\begin{aligned}
E_{cb}(L) &= \mathcal{T}^{00(11)}(L) \\
&= \frac{8}{(2\pi)^2} \sum_{l_0,l_1,l_2,l_3=0'}^{\infty} (-1)^{l_1+l_2+l_3+N_c} \left\{ \frac{2}{[\sum_{i=1}^3 (2Ll_i)^2]^2} \right\}
\end{aligned}
\tag{20}
$$

and

$$
\begin{aligned}
P_{cb}(L) &= \mathcal{T}^{33(11)}(L) \\
&= \frac{-8}{(2\pi)^2} \sum_{l_1,l_2,l_3=0'}^{\infty} (-1)^{l_1+l_2+l_3+N_c} \left\{ \frac{2\sum_{i=1}^3 (2Ll_i)^2 - 8(2Ll_3)^2}{[\sum_{i=1}^3 (2Ll_i)^2]^3} \right\}.
\end{aligned}
\tag{21}
$$

As another possibility for compactification, let calculate the expression for the Casimir pressure at zero temperature, considering the confinement in two dimensions (four plates), that is the Casimir effect for a wave guide, with $L_1 \to \infty$. In this case, using Eq.(18), we have the Casimir pressure, $P_{cg}(L)$, along the direction z (we consider the confinement in y- and z- axes, with $L_2 = L_3 = L$)

$$
\begin{aligned}
P_{cg}(L) &= \frac{8}{(2\pi)^2} \sum_{l_2,l_3=0'}^{\infty} (-1)^{l_2+l_3+N_c} \left\{ 2\frac{(2L_2l_2)^2 - 3(2L_3l_3)^2}{[(2L_2l_2)^2 + (2L_3l_3)^2]^3} \right\} \\
&= \frac{8}{(2\pi)^2} \sum_{l_2=1}^{\infty}\sum_{l_3=1}^{\infty} \left\{ (-1)^{l_2}(-1)^{l_3}\frac{2((2L_2(l_2))^2 - 6(2L_3(l_3))^2}{((2L_2(l_2))^2 + (2L_3(l_3))^2)^3} \right\} \\
&\quad - \frac{8}{(2\pi)^2} \sum_{l_2=1}^{\infty} \left\{ (-1)^{l_2}\frac{2((2L_2(l_2))^2}{((2L_2(l_2))^2)^3} \right\} + \frac{8}{(2\pi)^2} \sum_{l_3=1}^{\infty} \left\{ (-1)^{l_3}\frac{6(2L_3(l_3))^2}{((2L_3(l_3))^2)^3} \right\}.
\end{aligned}
\tag{22}
$$

The consistency of this formula can be verified by taking limits. For instance, from the Casimir pressure for the box, Eq.(21), we can recover Eq.(21) or Eq.(17) by taking, respectively, the limits $L_1 \to \infty$ and $L_1, L_2 \to \infty$.

In Figure 1 the full line is the plot of $P_c(L)$, the Casimir pressure due to two parallel plates separated by a distance L as given in Eq.(17); the dashed line is the Casimir pressure $P_{cb}(L)$ for a cubic box as given in Eq.(21) with $L_1 = L_2 = L_3 = L$ (the plot of Casimir pressure, $P_{cg}(L)$, for a wave guide obtained from Eq.(22) with $L_1 \to \infty$, $L_2 = L_3 = L$, is virtually the same as $P_c(L)$ in the scale of Figure 1). In Figure 2 there is the plot of $P_{cg}(L)$ derived from Eq.(21) with $L_1 \to \infty$, and $L_3 = L, L_2 = 0.1L$. Observe that the Casimir pressure is always positive in this case.

V. CONCLUDING REMARKS

In this paper we have presented a generalization of the Bogoliubov transformation in order to describe a massless fermion field compactified in an N-dimensional box at finite temperature. We write the (traceless) energy-momentum tensor from which we calculate and compare explicit expressions for the Casimir pressure, corresponding to different cases of confinement. The Casimir pressure for the case of two parallel plates and the cubic box are negative, imposing then an attractive force on the plates along the direction of the analysis. However, in the case for different rates among the sizes of the box, as in the example of the wave guide (the confinement in two directions) treated in Figure 2, the pressure is positive, representing a repulsive force among the plates. This repulsive force can have a direct influence in the description of quark deconfinement, pointing to an adverse effect when we compare it with the usual calculation of the Casimir force using the two plates (confinement along one dimension only), in which the pressure is attractive.

Another aspect, worthy of noting, is the simplicity of calculations, that can be observed from the known results for the fermion Casimir effect; see for instance [16,40]. This is so since we have avoided usual procedures, as the intricate method based on the sum of the quantum modes of the fields, satisfying some given boundary conditions. Indeed, instead of the sum of modes, we have used the Bogoliubov transformation to define the Casimir effect as a kind of condensation procedure of the fermion field (in a similar fashion as was carried out for the case of bosons [1]). Taking advantages of these practical proposals, the method developed here can be useful for calculations involving other geometries such as spherical or cylindrical symmetries. This analysis will be developed in more detail elsewhere.

REFERENCES

[1] J. C. da Silva, F.C. Khanna, A. Matos Neto and A. E. Santana, Phys. Rev. A **66** (2002) 052101.

[2] H. Umezawa, *Advanced Field Theory: Micro, Macro and Thermal Physics* (AIP, New York, 1993).

[3] Y. Takahashi and H. Umezawa, Coll. Phenomena **2** (1975) 55 (Reprinted in Int. J. Mod. Phys. 10 (1996) 1755).

[4] H. Umezawa, H. Matsumoto and M. Tachiki, *Thermofield Dynamics and Condensed States* (North-Holland, Amsterdan, 1982).

[5] I. Ojima, Ann. Phys. (N. Y.) **137** (1981) 1.

[6] A. E. Santana, A. Matos Neto, J. D. M. Vianna and F. C. Khanna, Physica A **280** (2000) 405.

[7] M. C. B. Abdalla, A. L. Gadelha and I. V. Vancea, Phys. Rev. D **64** (2001) 086005.

[8] M. C. B. Abdalla, A. L. Gadelha and I. V. Vancea, Int. J. Mod. Phys. A **18** (2003) 2109.

[9] H. B. G. Casimir, Proc. Ned. Akad. Wet. B **51** (1948) 793.

[10] P. W. Milonni, *The Quantum Vacuum* (Academic, Boston, 1993).

[11] V. M. Mostepanenko and N.N. Trunov, *The Casimir Effect and its Applications* (Clarendon, Oxford, 1997).

[12] M. Bordag, U. Mohideen and V. M. Mostepanenko, *New Developments in Casimir Effect*, quant-ph/0106045, Phys. Rep. **353** (2001) 1.

[13] F. S. Levin and D.A. Micha (Eds.), *Long Range Casimir Forces* (Plenum, New York, 1993).

[14] C. Seife, Science **275** (1997) 158.

[15] T. H. Boyer, Am. J. Phys. **71** (2003) 990.

[16] K. A. Milton, *The Casimir Effect: Physical Manifestations of Zero Point Energy*, hep-th/9901011.

[17] G. Plunien, B. Müeller and W. Greiner, Phys. Rep. **134** (1986) 87.

[18] M. Bordag (Ed.), *The Casimir Effect 50 Years Later* (World Scientific, Singapore, 1999).

[19] F. Caruso, N. P. Neto, B. F. Svaiter and N. F. Svaiter, Phys. Rev. D **43** (1991) 1300.

[20] F. Caruso, R. de Paola and N. F. Svaiter, Int. J. Mod. Phys. A **14** (1999) 2077.

[21] M. V. Cougo-Pinto, C. Farina and A. Tenório, Braz. J. Phys. **29** (1999) 371.

[22] S. K. Lamoreaux, Am. J. Phys. **67** (1999) 850.

[23] U. Mohideen and A. Roy, Phys. Rev. Lett. **81** (1998) 4549.

[24] O. Kenneth, I. Klich, A. Mann and M. Revzen, Phys. Rev. Lett. **89** (2002) 033001.

[25] T. Maruyama, K. Tsushima and A. Faessler, Nucl. Phys. A **537** (1992) 303.

[26] F. Serry, D. Walliser and G. J. Maclay, J. Appl. Phys. **84** (1998) 2501.

[27] E. Buks and M. L. Roukes, Phys. Rev. B **63** (2001) 033402.

[28] E. M. Lifshitz, Sov. Phys. JETP **2** (1956) 73.

[29] I. E. Dzyaloshinskii, E. M. Lifshitz and L. P. Pitaevskii, Adv. Phys.**10** (1961) 165.

[30] J. Mehra, Physica, **37** (1967) 145.

[31] G. L. Klimchitskaya and V. M. Mostepanenko, Phys. Rev. A **63** (2001) 062108.

[32] M. Revzen, R. Opher, M. Opher and A. Mann, Europhys. Lett. **38** (1997) 245 (1997).

[33] M. Revzen, R. Opher, M. Opher and A. Mann, J. Phys. A: Math and Gen. **30** (1997)

7783.

[34] M. Revzen and A. Mann, *Casimir Effect - The Classical Limit*, quant-ph/9803059.

[35] J. Feinberg, A. Mann and M. Revzen, Ann. Phys.(NY) 288 (2001) 103.

[36] L. S. Brown and G. J. Maclay, Phys. Rev. **184** (1969) 1272.

[37] D. Robaschik, K. Scharnhorst and E. Wieczorek, Ann. Phys.(N.Y.) **174** (1987) 401.

[38] S. Tadaki and S. Takagi, Prog. Theor. Phys. **75** (1982) 262.

[39] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. D **9** (1974) 3471.

[40] K. Saito, Z. Phys. C **50** (1991) 69.

[41] S. A. Gundersen and F. Ravndal, Ann. Phys. (N.Y.) **182** (1988) 90.

[42] C. A. Lütken and F. Ravndal, J. Phys. A: Math. Gen. **21** (1988) L793.

[43] C. A. Lütken and F. Ravndal, J. Phys. G: Nucl. Phys. **10** (1984) 123.

[44] F. Ravndal and D. Tollefsen, Phys. Rev. D **40** (1989) 4191.

[45] R. D. M. De Paola, R. B. Rodrigues and N. F. Svaiter, Mod. Phys. Lett. A **14** (1999) 2353.

[46] E. Elizalde, F. C. Santos and A. C. Tort, Int. J. Mod. Phys. A **18** (2003) 1761, hep-th/0206114.

[47] K. Johnson, Acta Phys. Pol. B **6** (1975) 865.

[48] L. A. Ferreira, A. H. Zimerman and J. R. Ruggiero, *Casimir Effect for Closed Cavities with Conducting and Permeable Walls,* Instituto de Física Teórica, UNESP, Preprint IFT-P-15/80.

[49] W. Lukosz, Physica **56** (1971) 109

[50] J. Ambjφrn and S. Wolfram, Ann. Phys. (N.Y.) **147** (1983) 1.

[51] J. Ambjφrn and S. Wolfram, Ann. Phys. (N.Y.) **147** (1983) 33.

[52] T. H. Boyer, Phys. Rev. A **9** (1974) 2078.

[53] J. C. da Silva, A. Matos Neto, H. Queiroz Plácido, M. Revzen and A. E. Santana, Physica A **292** (2001) 411.

[54] A. P. C. Malbouisson and J. M. C. Malbouisson, J. Phys. A: Math. Gen. **35** (2001) 2263.

[55] A. P. C. Malbouisson, J. M. C. Malbouisson and A. E. Santana, Nucl. Phys. B **631** (2002) 83.

[56] C. Itzykson and J. B. Zuber, *Quantum Field Theory (*McGrow-Hill, New York, 1980).

[57] K. Soutome, Z. Phys. C **40** (1988) 479.

Figure 1

Full line: Casimir pressure, P_c, for two parallel plates separeted by a distance L;dashed line: Casimir pressure, P_{cb}, for a box with sizes specifyed by $L_1 = L_2 = L_3 = L$

Figure 2

Casimir pressure, P_{cg}, for a wave guide obtained from a box with $L_1 \to \infty$, and $L_3 = L, L_2 = 0.1L$